Physics 371 Fall 2020 Prof. Anlage
Review

Quantum Physics

JJ Thomson charge-to-mass measurement in E, B fields: $q/m = \frac{v}{RB}$.
Millikan oil droplet experiment: revealed the quantization of electric charge.

Blackbody radiation, Stefan-Boltzmann law: $R_{Total} = \sigma T^4$, $\sigma = 5.6703 \times 10^{-8} \frac{W}{m^2 K^4}$.
Wien displacement law says that $\lambda_{max} T = 2.898 \times 10^{-3} m - K$.
Radiation power per unit area related to the energy density of a blackbody: $R(\lambda) = \frac{c}{4}\rho(\lambda)$.
Rayleigh-Jeans (classical equipartition argument) law $\rho(\lambda) = 8\pi k_B T/\lambda^4$ leads to the 'ultraviolet catastrophe'.
Planck blackbody radiation (treat the atoms as having discrete energy states, and the light as having energy $E = hf$): $\rho(\lambda) = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda k_B T} - 1}$, $h = 6.626 \times 10^{-34} J - s$.

Photoelectric effect and the concept of light as a particle (photon with $E = hf$): $hf = eV_0 + \phi$. Photon collides with one electron and transfers all of its energy, $-V_0$ is the stopping potential.

X-ray production by Bremsstrahlung with cutoff $\lambda_{min} = \frac{1240}{V}\, nm$ (Duane-Hunt Rule), explained by Einstein as inverse photoemission with $\lambda_{min} = \frac{hc}{eV}$. Sharp emission lines arise from quantized energy levels in the 'core shells' of atoms.
Bragg reflection of x-rays from layers of atoms in crystals: $n\lambda = 2d \sin\theta$, where $n = 1, 2, 3, \dots$, d is the spacing between the parallel layers.

Rutherford scattering (Phys 410) suggested that positive charge is concentrated in a very small volume – the nuclear model of the atom.
Empirical rule for light emission from hydrogen $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$, Rydberg constant $R = R_H = 1.096776 \times 10^7 \frac{1}{m}$ for Hydrogen.
Bohr model of the hydrogen atom (assumes stationary states, light comes from transitions between stationary states, electron angular momentum in circular orbits is quantized): $|\vec{L}| = |\vec{r} \times m\vec{v}| = mvr = n\hbar$, with $n = 1, 2, 3, \dots$, Radius of circular orbits: $r_n = \frac{n^2 a_0}{Z}$ with $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = 0.529$ Å, Total energy of Hydrogen atom: $E_n = -E_0 \frac{Z^2}{n^2}$, with $E_0 = \frac{mc^2 (e^2/4\pi\varepsilon_0)^2}{2\,(\hbar c)^2} = \frac{mc^2}{2}\alpha^2 = 13.6\, eV$, $\alpha = \frac{e^2/4\pi\varepsilon_0}{\hbar c} \cong \frac{1}{137}$ is called the 'fine structure constant'. Explains the Hydrogen atom emission spectrum but not multi-electron atoms.

Davisson-Germer experiment shows that matter (electrons) diffract from periodic structures (Ni atoms on a surface) like waves. It is clear that matter has a strong wave-like character when measured under appropriate conditions.

deBroglie proposed the wavelength of matter waves as $\lambda_{dB} = h/p$, where p is the linear momentum. Classical physics should be recovered in the short-λ_{dB} limit – the Correspondence Principle

The time-dependent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$;
Separation of variables leads to $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (a property of stationary states);
Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\,\psi(x) = E\psi(x)$;
The wavefunction $\Psi(x,t)$ is complex in general and cannot be measured. Born interpretation of the wave function in terms of a probability density $P(x,t) = \Psi^*(x,t)\Psi(x,t)$; Only real and finite quantities can be measured experimentally.
Normalization condition: $\int_{-\infty}^{+\infty}|\psi(x)|^2\,dx = 1$.

Particle of mass m in an **infinite square well** between $x = 0$ and $x = L$: $E_n = \frac{\hbar^2 {k_n}^2}{2m} = n^2\frac{\pi^2\hbar^2}{2mL^2}$ with $n = 1, 2, 3, \ldots$, and $\psi_n(x) = \sqrt{2/L}\ \sin k_n x$.
Finite square well of height V_0, energy eigenvalues are solutions of the transcendental equation: $\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{V_0-E}{E}}$ (even parity solutions). Always at least one solution!
Harmonic oscillator: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}m\omega^2x^2\psi(x) \;= E\psi(x)$, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0, 1, 2, 3, \ldots$, Eigenfunctions: $\psi_n(x) = C_n\, e^{-m\omega^2x^2/2\hbar}\, H_n(x)$, involve the Hermite polynomials multiplying a Gaussian in x.
Classical turning points are inflection points in $\psi(x)$.

Step potential $V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$ has reflection rate $R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$, and transmission rate $T = \frac{4\,k_1\,k_2}{(k_1+k_2)^2}$, where $k_1 = \sqrt{2mE}/\hbar$ and $k_2 = \sqrt{2m(E - V_0)}/\hbar$.
Tunneling probability through barrier $T = \left[1 + \frac{\sinh^2(\alpha a)}{4\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}\right]^{-1} \approx 16\ \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a}$, where a is the barrier width, and $\alpha = \sqrt{2m(V_0 - E)}/\hbar$.

General wave uncertainty properties: $(\Delta x)\,(\Delta k) \geq 1/2$, $(\Delta t)\,(\Delta\omega) \geq 1/2$.
Quantum uncertainty properties: $(\Delta x)\,(\Delta p) \geq \hbar/2$, $(\Delta t)\,(\Delta E) \geq \hbar/2$.

Spin Angular Momentum: A "two-valudeness not describable classically." Spin quantum number $s = 1/2$, with $m_s = \pm 1/2$. Stern-Gerlach device.

Hydrogen Atom 3D TISE: $\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r}) + \mathrm{V}(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$. In spherical coordinates: $\frac{-\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r,\theta,\phi)}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi(r,\theta,\phi)}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2\psi(r,\theta,\phi)}{\partial\phi^2}\right)\right] + \mathrm{V}(r,\theta,\phi)\psi(r,\theta,\phi) = E\,\psi(r,\theta,\phi)$ Consider central forces only, such that $V = V(r)$ only. Separate variables as $\psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$ to arrive at two new equations:

Radial $\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)-\frac{2mr^2}{\hbar^2}[V(r)-E]R=\ell(\ell+1)R$ and Angular $\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\left(\frac{\partial^2 Y}{\partial\phi^2}\right)\right]=-\ell(\ell+1)Y$.

Solution to the angular equation are the spherical harmonics: $Y_\ell^m(\theta,\phi)$ with $\ell\geq 0$, and $|m|\leq\ell$. The length squared of $\vec{L}$ is given by $\ell(\ell+1)\hbar^2$. The z-component of $\vec{L}$ is given by $m\hbar$.

Solution to the radial equation gives the Principal quantum number n and the energies $E_n=-\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\frac{1}{n^2}$, where $n=1,2,3,\ldots$, $E_1=-13.6\,eV$. Bohr radius: $a=\frac{4\pi\varepsilon_0\hbar^2}{me^2}=0.529\times10^{-10}\,m$.

Full solution: $\psi_{n\ell m}(r,\theta,\phi)=\sqrt{\left(\frac{2}{na}\right)^3\frac{(n-\ell-1)!}{2n\,(n+\ell)!}}\left(\frac{2r}{na}\right)^\ell L_{n-\ell-1}^{2\ell+1}\left(\frac{2r}{na}\right)e^{-r/na}\,Y_\ell^m(\theta,\phi)$

$n=1,\ 2,\ 3,\ \ldots$ Principal Quantum Number (an infinite number of bound states)
$\ell=0,\ 1,\ 2,\ 3,\ \ldots n-1$ Angular Momentum Quantum Number (n values)
$m_\ell=-\ell,\ -\ell+1,\ \ldots,\ 0,\ \ldots,\ \ell-1,\ \ell$ Magnetic Quantum Number ($2\ell+1$ values)
$m_s=-\frac{1}{2},\ +\frac{1}{2}$ Spin Quantum Number (2 values)

Hence a quantum state of the hydrogen atom is specified by a list of 4 quantum numbers: (n,ℓ,m_ℓ,m_s). Degeneracy of the nth state of the hydrogen atom is given by $d(n)=2\sum_{\ell=0}^{n-1}(2\ell+1)=2n^2$

Multi-Electron Atoms: Utilize the Independent Particle Approximation (Mean Field Theory) to find an effective potential for any particular electron that allows a Hydrogen-like solution for the electron states.

Nuclear Physics: The nucleus is small and approximately spherical with a radius $R=R_0A^{1/3}$, where $R_0=1.07\,fm$. The nucleus is a bound state of Z protons and N neutrons with binding energy B defined as $m_{nucleus}=Z\,m_{proton}+N\,m_{Neutron}-\frac{B}{c^2}$. The notation for identifying a specific nucleus is as follows: ${}_Z^AX_N$, where X is a code name for Z from the periodic table, and $A=Z+N$. The density of the nucleus is constant to good approximation.

Binding energy $B\approx a_{vol}A-a_{surf}A^{\frac{1}{3}}-a_{Coul}\frac{Z^2}{A^{1/3}}$ shows corrections due to surface effects and Coulomb repulsion giving rise to a peak value of B/A at ${}_{26}^{56}Fe_{30}$.

Radioactivity: r = probability that a nucleus will decay in a unit time period. $N(t)=N_0e^{-rt}$. Lifetime $\tau=1/r$. Half-life $t_{1/2}=\tau\,ln2$. $N(t)=N_0/2^{t/t_{1/2}}$.

Alpha decay of heavy nuclei: $lnt_{1/2}=\frac{aZ}{\sqrt{K}}-b\sqrt{Z\,R}+c$

What is Important? Quantum mechanics is a *wave theory of matter*. You should not impose classical notions on the solutions to quantum problems.

The Bohr model of the hydrogen atom – a good starting point!

Solving the TISE quickly and efficiently and accurately

Properties of the <u>ubiquitous</u> QM problems:

Infinite Square Well, Harmonic Oscillator, Free Particle, Finite square well, scattering problems

Energy values, wavefunctions, quantum numbers and their possible values and constraints, degeneracies

Being able to sketch wavefunctions for new potentials using intuition

Problem-Solving strategies: Go back to the Schrodinger equation and the interpretation of the wavefunction. In quantum mechanics there is no information available beyond that you can derive from the wavefunction.